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Extensions to the Analysis of Choice Studies

Thomas L. Pilon,
TRAC, Inc.
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Extensions to the Analysis of Choice Studies

Thomas L. Pilon[1]
TRAC, Inc.

Most choice studies have made use of "standard" analysis, without attention to differential cross elasticities or unequal competitive effects among brands. This paper will present results from a large client-sponsored data set demonstrating how suitably designed choice studies can also be used to measure differential cross effects among brands. This can lead to more accurate simulators of market behavior, as well as "maps" which graphically portray the extent of competition among brands.

Choice vs. Conjoint

Conventional conjoint analysis may lead to biased estimates of price sensitivity. In particular, price sensitivity may be systematically understated (Luery, 1990). Also, most types of conjoint analysis are limited in terms of the number of brands or SKUs that are included in the study. In the beer study that is described below, there were 42 brands included. Also, there were five major pack types for most brands: 6-pack cans, 6-pack bottles, 12-pack cans, 12-pack bottles, and cases of 24 cans. Furthermore, there would have been many more brands and pack types (cases of 24 bottles, 18 packs, 30 packs, etc.) if the budget would have allowed it. In recent years, the marketing research community has discovered that these limitations of conventional conjoint analysis can be circumvented through the use of choice studies. In fact, it is probably appropriate to say the choice based conjoint analysis is the “tool of choice” for pricing studies in the mid-nineties.

It is the purpose of this paper to discuss and show a few extensions to the standard analysis of choice data. The extensions all have to do with the derivation and analysis of cross effects, also known as cross “elasticities”. One extension is the derivation of the cross effect matrix itself. Another extension involves rescaling the cross effect matrix so that it can be portrayed in a multidimensional scaling type map. A third extension shows the improvements to standard conjoint simulators that result when cross effects are included in the simulator.

Before describing the data and the choice study, definitions of elasticities and cross-elasticities and a brief review of the marketing literature will be provided.

[1] The author wishes to acknowledge both theoretical and computational contributions from Bryan Orme and Rich Johnson, both of Sawtooth Software.

Elasticities and Cross-Elasticities - Background

It has long been established in the economic literature that the price actions of one product (or brand) affect the sales (or share) of other products. Econometricians refer to this as "degree of substitutability." This substitutability can be quantified in terms of "elasticities" and "cross-elasticities. A price elasticity (hereinafter referred to as elasticity) can be expressed algebraically as:

$$\frac{\%\Delta S_A}{\%\Delta P_A}$$

where the numerator is the change (Greek letter delta) in sales for brand A, S_A, as a percentage of the original sales volume of brand A and the denominator is the change in price of brand A, P_A, as a percentage of the original price of brand A.

A price cross elasticity (hereinafter referred to as cross elasticity) can be expressed algebraically as:

$$\frac{\%\Delta \mathrm{S_B}}{\%\Delta \mathrm{P_A}}$$

where the numerator is the change in sales volume for brand B as a percentage of the original sales of brand B and the denominator is the change in price of brand A as a percentage of the original price of brand A.

The use of price elasticities and cross-elasticities in pricing studies is not new as summarized by Rao (1984) in a review of over 200 pricing studies. Recent scholarly articles in this domain include Reibstein and Gatignon (1984) who demonstrated the importance of using elasticities and cross elasticities in product line pricing, and Cooper (1988) who used maps to portray how brands influence competing brands (more on this later). Other recent studies include Krishnamurthi, Raj, and Sivakumar (1995), Cooper, Klapper and Inoue (1996) and Guiltinan and Gunlach (1996), Gupta, Chintagunta, Kaul, and Wittink (1996) and Richard, Allaway, Berkowitz and D'Souza (1996).

Discussions of price elasticity models have appeared in the practitioner literature as well. Luery (1990) describes the evolution of conjoint analysis and the use of cross-effects in simulators. Smallwood (1991), Datoo (1994) and Mohn (1995) all provide easy to understand introductions to the uses of price elasticities and cross elasticities in choice and conjoint models. Wyner, Benedetti and Trapp (1984) provide a very readable paper on price elasticity choice models.

Any review of the applied pricing literature would be remiss without mention of Nagle (1987) and Monroe (1990); both provide excellent comprehensive discussions of applied pricing. Finally, for more rigorous discussions of pricing issues see Devinney (1988) and for an advanced discussion of market response models see Hanssens, Parsons, and Schultz (1990).

The Beer Study

In 1994, a beer manufacturer commissioned a study to learn more about the effects of pricing in their industry. Although the study was conducted in 10 major markets in the U.S., the data presented below are from one market only. The name of that market will remain unmentioned for proprietary reasons. Over 1,400 choice interviews were conducted. On average, each respondent completed slightly less than 20 choice tasks; thus the data consists of nearly 28,000 choice tasks.

The data were collected using Sawtooth Software's Ci3 computerized interviewing program. Due to the large number of brands (42) and other complexities of the study, it was not possible to use Sawtooth Software's Choice Based Conjoint program. Respondents were asked to choose five brands from a list of 42 brands that they would most likely buy or consider buying in a certain situation at a certain type of outlet. Based on these selections, Ci3 then configured a choice screen (see Appendix A - Ci3 Choice Screen 1 for an example). Given a matrix of these brands and pack types and randomly chosen prices within each combination of brand and pack type, respondents were asked which brand they would choose. After all the information about the other brands was removed from the screen, respondents were asked which pack type they would choose (see Appendix A - Ci3 Choice Screen 2). Finally, after choosing the pack type, respondents were asked how many units of that brand/pack type they would purchase at the price that was shown (see Appendix A - Ci3 Choice Screen 3). The combination of the three screens described above represented one task.

Elasticities and Cross-Elasticities

To calculate the elasticities, the log of the number of units of a brand chosen (adjusted for pack type size) was regressed against the log of the brand's price. The double-log transformation is commonly employed by econometricians (Johnston, 1984) because it corresponds to the assumption of a constant elasticity between brand and price and the simple application of linear methods to the logarithms of the variables directly produces an estimate of that elasticity (the ∃s are the elasticities). The cross elasticities were calculated in the same manner: the log of the number of units of a brand chosen was regressed against the log of the competitive brand's price. Of course, only tasks that included both brands could be used in this case.

The elasticities and cross-elasticities were each calculated independently in a series of bivariate regressions. Since the Ci3 was randomized, and therefore essentially orthogonal, the effects could be investigated separately without loss of information. Also, trying to estimate all the effects in one model would have required too many coefficients to estimate reliably at one time.

Running the 144 (12 elasticity and 132 cross-elasticity) regressions yield the cross elasticity matrix shown in Table 1. The diagonal elements are individual brand elasticities. Elasticities are usually negative because price and choice volume typically move in opposite directions within a given brand (a brand decreases its price and its choice volume increases). For example, the elasticity of Miller Lite (MIL) is -2.10, thus, if Miller Lite increases its price by 1 percent, its choice volume would drop by 2.10 percent (assuming all other things were held constant, which of course, they never are). The off-diagonal elements are cross elasticities.

Cross-elasticities are usually positive because the price of one brand and the choice volume of competing brands typically move in the same direction (a brand decreases its price and the choice volume of competing brands decreases). For example, the 1.18 in the Budweiser (Bud) row and the Coors column indicates that if the price of Bud was increased by 1 percent, Coors choice volume would increase by 1.18 percent.

Table 1 - Cross Elasticity Matrix

	Bud	**BudL**	**Mich**	**MichL**	**MGD**	**MGDL**	**MlL**	**Coors**	**CoorL**	**Hein**	**Molsn**	**Sam**
Bud	**-3.80**	1.70	1.87	1.11	1.68	0.50	0.70	1.18	0.61	0.90	0.60	0.60
BudL	1.50	**-3.34**	1.30	1.60	1.10	0.77	0.97	0.80	1.00	0.90	0.40	0.51
Mich	1.10	0.68	**-3.18**	1.00	0.92	0.29	0.38	0.74	0.35	0.12	0.23	0.56
MichL	0.70	1.14	0.92	**-3.53**	0.46	0.52	0.63	0.33	0.72	0.08	0.19	0.51
MGD	0.90	0.70	0.97	0.50	**-3.42**	0.96	0.45	0.73	0.42	0.09	0.22	0.64
MGDL	0.20	0.37	0.19	0.33	0.64	**-2.20**	0.32	0.18	0.29	0.04	0.14	0.31
MlL	0.60	1.26	0.66	1.32	0.90	0.92	**-2.10**	0.54	1.07	0.12	0.17	0.32
Coors	0.70	0.44	0.81	0.37	0.83	0.27	0.33	**-2.30**	0.54	0.06	0.18	0.45
CoorL	0.40	1.27	0.55	1.30	0.75	0.71	1.02	0.80	**-2.20**	0.14	0.13	0.51
Hein	0.60	0.62	0.85	0.86	0.75	0.41	0.46	0.50	0.52	**-0.42**	0.37	1.18
Molsn	0.40	0.34	0.44	0.40	0.53	0.38	0.18	0.27	0.18	0.09	**-0.96**	0.77
Sam	0.29	0.22	0.34	0.35	0.40	0.31	0.11	0.26	0.19	0.09	0.23	**-2.85**

It is not surprising to see the relatively large elasticities in the Bud and Bud Light rows because they are by far the two largest-selling brands (see Table 3 below). Suppose that there are just two brands in the market place. Brand L sells 100,000 units a year and Brand S sells 50,000 units a year. If Brand L lowers its price and increases its sales by 1 percent (gains 1,000 units), and the smaller brand does not react (and the market is in some type of equilibrium), then Brand S will lose a 1,000 units in sales--but 1,000 units is 2 percent of Brand S's sales.

Product Maps

While cross-elasticity matrices are informative, they do not offer ready access to the big picture of the relative price sensitivities among brands. Careful study of the cross-elasticity matrix, along with a subjective adjustment for the size effect discussed above, reveals that the "most similar" brands have the largest cross-elasticities. Both the desire to see the "big picture" and the difficulty of subjectively adjusting elasticities inspired the development of an algorithm that would remove brand size effects and rescale the matrix so that it was amenable to the arsenal of mapping techniques that market researchers have developed.

The effect of brand size can be removed from the elasticities so that the "elasticities" are not percentage changes, but instead are proportional to absolute changes. Rescaling the rows of the matrix to be of the same size removes the effect of the size of the "active" brand (brand making the price changes). Rescaling the columns of the matrix to be of the same size removes the effect of the size of the "passive" brand (brands affected by the price change). Iteratively rescaling the rows and columns until they converge removes both effects and makes the final result independent of whether the rows or columns were rescaled initially. After the process converges, the matrix is much more symmetric, but not exactly so. Because most mapping programs require a symmetric matrix as input, the elements on each side of the diagonal were averaged[2].

Table 2 - Similarities Matrix

	Bud	**BudL**	**Mich**	**MichL**	**MGD**	**MGDL**	**MlL**	**Coors**	**CoorL**	**Hein**	**Molsn**	**Sam**
Bud	**-1.00**	0.45	0.42	0.25	0.34	0.11	0.23	0.31	0.17	0.68	0.26	0.13
BudL	0.45	**-1.00**	0.29	0.40	0.26	0.20	0.42	0.21	0.42	0.76	0.21	0.11
Mich	0.42	0.29	**-1.00**	0.29	0.29	0.09	0.19	0.29	0.17	0.28	0.18	0.13
MichL	0.25	0.40	0.29	**-1.00**	0.14	0.15	0.34	0.12	0.35	0.23	0.15	0.13
MGD	0.34	0.26	0.29	0.14	**-1.00**	0.29	0.24	0.28	0.20	0.23	0.19	0.16
MGDL	0.11	0.20	0.09	0.15	0.29	**-1.00**	0.25	0.10	0.21	0.14	0.16	0.12
MlL	0.23	0.42	0.19	0.34	0.24	0.25	**-1.00**	0.19	0.49	0.26	0.12	0.08
Coors	0.31	0.21	0.29	0.12	0.28	0.10	0.19	**-1.00**	0.29	0.19	0.15	0.13
CoorL	0.17	0.42	0.17	0.35	0.20	0.21	0.49	0.29	**-1.00**	0.29	0.11	0.12
Hein	0.68	0.76	0.28	0.23	0.23	0.14	0.26	0.19	0.29	**-1.00**	0.30	0.30
Molsn	0.26	0.21	0.18	0.15	0.19	0.16	0.12	0.15	0.11	0.30	**-1.00**	0.26
Sam	0.13	0.11	0.15	0.13	0.16	0.12	0.08	0.13	0.12	0.30	0.26	**-1.00**

Finally, the matrix was "standardized" by dividing each element by the square root of the product of the diagonal elements. This "standardization" removes the arbitrary scaling so that products have unit similarity with themselves. The final similarities matrix is presented in Table 2. See Appendix B for the details of the step by step process of getting from the cross-elasticity matrix to the similarities matrix.

Figure 1 - MDS Map of Similarities Data in Table 2

Again, the goal of the above exercise was to rescale the cross elasticity matrix into a matrix from which we could produce a map that shows the relative degree of price sensitivity among brands. The similarities matrix in Table 2 can now be subjected to various types of metric and non-metric multi-dimensional scaling techniques. The map in Figure 1 was produced using the Systat Multidimensional Scaling (MDS) routine which is non-metric. A Kruskal loss function that produces results comparable to the well known Bell Labs' KYST was used. Seventy-six percent of the variance is explained by the two dimensions. Examination of the

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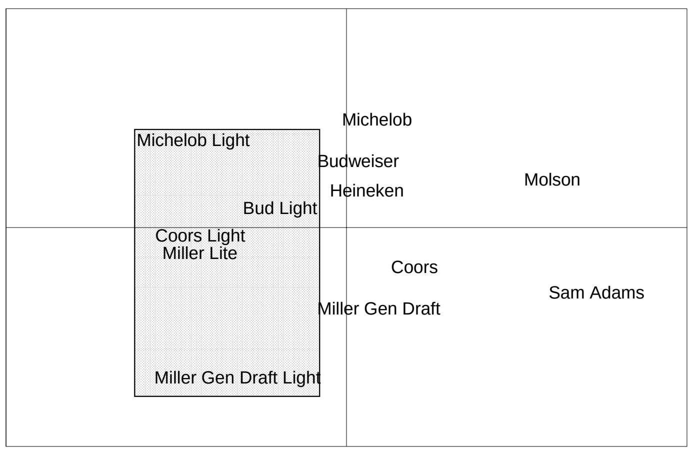
[2] Using a special case of three mode factor analysis on the original asymmetric cross elasticity matrix, Cooper (1988) was able to derive two sets of brand positions, one which portrays how brands exert influence over the competition and the other which portrays how brands are influenced by others.

Shepard Diagram, which is a scatterplot of distances between points in the MDS plot versus the similarities that were input, indicated that there was a good fit.

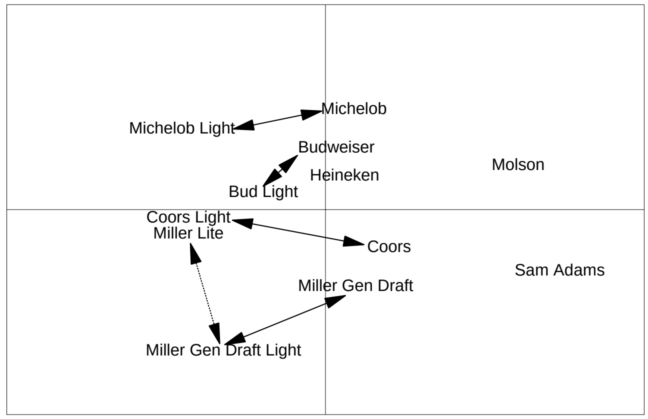
The Young loss function, which is designed to produce results comparable to ALSCAL (available in SPSS Professional Statistics 7.5), produced very similar results. With the assumption that there is a linear relationship between distances and similarities, a principal components or factor analysis method could have been used. Note, for methods that require a full matrix (such as principal components), the signs of the diagonals should be reversed. Usually, but not necessarily, multidimensional scaling can fit an appropriate model in fewer dimensions than can principal components, so MDS was chosen. See Pilon (1989, 1992) for applied comparisons of results obtained from alternative perceptual mapping techniques or see Green, Carmone, and Smith (1989) for a much more detailed discussion. Also, the chapter on perceptual mapping in Hair, Anderson, Tatham and Black (1995) contains an excellent readable discussion.

Figure 2 - MDS Map with Light Beers in Shaded Box



The map has face validity in that all of the light beers are together (see Figure 2). Also, Figure 3 shows that all the pairs of companion brands (the regular and light beers of the same brand) are relatively close to together. Miller Lite is not really a companion brand to Miller Genuine Draft and Miller Genuine Draft Light but it is in the “Miller” neighborhood. In general, the horizontal axis can be interpreted as a “lightness/ heaviness” dimension while the vertical axis can be interpreted as a “manufacturer” dimension.

Figure 3 - MDS Map with Companion Brands Connected



If these maps can be believed, they have very important pricing ramifications. Seemingly, brands may compete with their companion brands as much as their competitive brands. When a brand decreases its price, it seems that it would take as many customers away from the companion brand as it would from its competitive brands.

Cross Elastic Simulators

With these observations in mind, a simulator was built that included these cross elastic effects. Most conjoint simulators, especially those derived from main effects only conjoint models, allocate the share given up by a brand that raises its price proportionately (to share) across all the other brands in the simulator. In many (if not most) cases, this proportionate allocation is not an accurate representation of how markets actually respond.

One of the major difficulties in conjoint analysis is overcoming the limitations of the independence of irrelevant alternatives (IIA) problem. The best explanation that I have seen of this problem is:

> The basic idea of IIA is that the ratio of any two products' shares should be independent of all other products. This sounds like a good thing, and at first, IIA was regarded as a beneficial property.
>
> However, another way to say the same thing is that an improved product gains share from all other products in proportion to their shares; and when a product loses share, it loses to others in proportion to their shares. Stated that way, it is easy to see that IIA implies an unrealistically simple model. In the real world, products compete unequally with one another, and when an existing product is improved, it usually gains most from a subset of products with which it competes most directly.
>
> Imagine a transportation market with two products, cars and red busses, each having a market share of 50%. Suppose we add a second bus, colored blue. An IIA simulator would predict that the blue bus would take share equally from the car and red bus, so that the total bus share would become 67%. But it's clearly more reasonable to expect that the blue bus would take share mostly from the red

bus, and that total bus share would remain close to 50%. Indeed, the IIA problem is sometimes referred to as the "red bus, blue bus problem." (Johnson, 1997)

By incorporating the cross elasticities from Table 1 above into a simulator, the IIA problem is greatly alleviated. In the Cross-Elasticity Simulator, the coefficients from each column of the Bud row of Table 1 were applied independently. Specifically, the volume of Bud was reduced by 3.8%, the volume of Bud Light was increased by 1.7%, ..., and the volume of Sam Adams was increased by 0.6%. As a final step, the resulting shares were rescaled to sum to 100. In the Standard IIA Simulator, only the elasticity of Bud was applied. The share that Bud gave up was allocated proportionately across the other brands' shares. Again, the resulting shares were rescaled to sum to 100. Table 3 shows how the results differ from a simulator that includes the main effects (elasticities) only. Note that the magnitude of Bud's percent loss is much less with the Standard IIA Simulator than with the Cross-Elasticity Simulator. Also, note the variation in the % Gain with the Cross-Elasticity Simulator as opposed to the constant % Gain with the Standard IIA Simulator. While "truth" is not known, the Cross-Elasticity Simulator results seem to coincide more with what one would expect. If one believes the cross elasticity matrix above, then one would believe the Cross Elasticity Simulator's results more so than the Standard IIA Simulator's results.

Table 3 - Simulation Results from a 1% increase in Bud's Price

	Base Case Market Share	Standard IIA Simulator Share	% Gain/ Loss	Cross Elasticity Simulator Share	% Gain/ Loss
Bud	31.80%	30.96%	-2.69%	30.73%	-3.46%
BudLgt	16.60%	16.80%	1.21%	16.96%	2.13%
Michelob	1.36%	1.37%	1.21%	1.39%	2.30%
MichelobL	1.72%	1.74%	1.21%	1.75%	1.56%
MillerGD	8.06%	8.16%	1.21%	8.23%	2.11%
MillerGDL	3.80%	3.84%	1.21%	3.83%	0.97%
MillerLite	17.63%	17.85%	1.21%	17.84%	1.16%
Coors	2.63%	2.66%	1.21%	2.67%	1.63%
CoorsLgt	14.90%	15.09%	1.21%	15.07%	1.07%
Heineken	0.85%	0.86%	1.21%	0.87%	1.36%
Molson	0.41%	0.41%	1.21%	0.41%	1.06%
SamAdm	0.24%	0.24%	1.21%	0.24%	1.06%
	100.0%	100.0%		100.0%	

Discussion

Choice studies like the one described above have several advantages over conjoint studies. Specifically, they allow for unique price levels and price effects (utilities) for each brand. As was shown above, they also allow for the calculation of cross-effect matrices. These cross effects can be graphically portrayed in various types of perceptual maps. When cross effects are incorporated into simulators, they yield more believable results than traditional simulators and they also alleviate the IIA problem that has plagued conjoint simulators since their inception.

However, one problem with this type of study is the focus is clearly on price. Although we tried to disguise the price focus of the study by varying the situation and outlet type, it did not take respondents long to realize that we were playing pricing games. We may have made them overly sensitive to price. It would have been better to have a few other attributes and fewer brands and pack types and price points for each brand/pack type.

Another problem with this type of study is that various types of statistical anomalies may occur. Cross-effects can be very small or negative requiring smoothing and repercentaging results of simulators so that they add to 100 can sometimes create reversals.

Finally, I think it would be very useful to find a simpler way than Cooper (1988) to show both how a brand is affected by other brands and how a brand affects others brands, rather than simply rescaling to remove the size factor and then averaging the two effects as was done above.

Other methods that are commonly used for pricing studies have problems, as well. Discrete Choice Models add other attributes and varying them across scenarios deflects the undue emphasis on price and decreases response bias, but bring back the IIA problem. Mother logit models do allow for cross-effects other than strictly proportionate share draws, but are complex and are not available to most researchers.

Appendix A - Ci3 Choice Screen 1

If you were buying beer at a Convenience Store/Gas Station for personal consumption at home with family members and these choices were available, which would you choose?

Press the key for that BRAND number.

	1 Budweiser	2 Bud Light	3 Coors	4 Miller Gen- uine Draft	5 Miller Gen. Draft Light
6-pack cans	$6.99	$5.99	$6.99	$5.99	$6.99
6-pk NR bottles	$3.49	$3.99	$4.99	$3.99	$3.99
12-pack cans	$5.99	$9.99	$5.99	$7.99	$5.99
12-pk NR bottles	$8.99	$11.99	$7.99	$11.99	$7.99
Case of 24 cans	$14.99	$10.99	$14.99	$14.99	$16.99

NR = Non-Returnable

Press 6 if you wouldn't buy any of these.

Press the key (1-6) to show your choice, or <ESC> if you want to back up.

Appendix A - Ci3 Choice Screen 2

If you were buying beer at a Convenience Store/Gas Station which Package Type of **Budweiser** would you choose?

Press the key for that PACKAGE TYPE number.

	Budweiser	
6-pack cans	$6.99	1
6-pk NR bottles	$3.49	2
12-pack cans	$5.99	3
12-pk NR bottles	$8.99	4
Case of 24 cans	$14.99	5

'NR' means Non-Returnable.
Press the key (1-5) to show your choice or <ESC> if you want to back up.

Appendix A - Ci3 Choice Screen 3

If you were buying beer at a Convenience Store/Gas Station for personal consumption at home with family members,

HOW MANY 12-packs of cans of Budweiser would you buy?

Type that number of Packages, then press <ENTER>.

Budweiser
12-packs of cans
$5.99

______ 12-packs of cans

Type the number you would buy and then press <ENTER>.
Press <ESC> if you want to back up.

Appendix B - Iterative Rescaling of Cross Elasticity Matrix to Similarities Matrix

Original Cross Elasticity Matrix:

	Bud	BudL	Mich	MichL	MGD	MGDL	MIL	Coors	CoorsL	Hein	Molsn	Sam	Off Diag Row Sum
Bud	-3.80	1.70	1.87	1.11	1.68	0.50	0.70	1.18	0.61	0.90	0.60	0.60	11.45
BudL	1.50	-3.34	1.30	1.60	1.10	0.77	0.97	0.80	1.00	0.90	0.40	0.51	10.85
Mich	1.10	0.68	-3.18	1.00	0.92	0.29	0.38	0.74	0.35	0.12	0.23	0.56	6.36
MichL	0.70	1.14	0.92	-3.53	0.46	0.52	0.63	0.33	0.72	0.08	0.19	0.51	6.19
MGD	0.90	0.70	0.97	0.50	-3.42	0.96	0.45	0.73	0.42	0.09	0.22	0.64	6.58
MGDL	0.20	0.37	0.19	0.33	0.64	-2.20	0.32	0.18	0.29	0.04	0.14	0.31	3.01
MIL	0.60	1.26	0.66	1.32	0.90	0.92	-2.10	0.54	1.07	0.12	0.17	0.32	7.89
Coors	0.70	0.44	0.81	0.37	0.83	0.27	0.33	-2.30	0.54	0.06	0.18	0.45	4.97
CoorsL	0.40	1.27	0.55	1.30	0.75	0.71	1.02	0.80	-2.20	0.14	0.13	0.51	7.57
Hein	0.60	0.62	0.85	0.86	0.75	0.41	0.46	0.50	0.52	-0.42	0.37	1.18	7.12
Molsn	0.40	0.34	0.44	0.40	0.53	0.38	0.18	0.27	0.18	0.09	-0.96	0.77	3.98
Sam	0.29	0.22	0.34	0.35	0.40	0.31	0.11	0.26	0.19	0.09	0.23	-2.85	2.79
Off Diag Col Sum:	7.39	8.74	8.89	9.14	8.95	6.04	5.55	6.32	5.90	2.63	2.86	6.35	

Rescale rows by dividing every element in the row by its row sum from the table above:

	Bud	BudL	Mich	MichL	MGD	MGDL	MIL	Coors	CoorsL	Hein	Molsn	Sam	Off Diag Row Sum
Bud	-0.33	0.15	0.16	0.10	0.15	0.04	0.06	0.10	0.05	0.08	0.05	0.05	1.00
BudL	0.14	-0.31	0.12	0.15	0.10	0.07	0.09	0.07	0.09	0.08	0.04	0.05	1.00
Mich	0.17	0.11	-0.50	0.16	0.14	0.05	0.06	0.12	0.05	0.02	0.04	0.09	1.00
MichL	0.11	0.18	0.15	-0.57	0.07	0.08	0.10	0.05	0.12	0.01	0.03	0.08	1.00
MGD	0.14	0.11	0.15	0.08	-0.52	0.15	0.07	0.11	0.06	0.01	0.03	0.10	1.00
MGDL	0.07	0.12	0.06	0.11	0.21	-0.73	0.11	0.06	0.10	0.01	0.05	0.10	1.00
MIL	0.08	0.16	0.08	0.17	0.11	0.12	-0.27	0.07	0.14	0.02	0.02	0.04	1.00
Coors	0.14	0.09	0.16	0.07	0.17	0.05	0.07	-0.46	0.11	0.01	0.04	0.09	1.00
CoorsL	0.05	0.17	0.07	0.17	0.10	0.09	0.13	0.11	-0.29	0.02	0.02	0.07	1.00
Hein	0.08	0.09	0.12	0.12	0.11	0.06	0.06	0.07	0.07	-0.06	0.05	0.17	1.00
Molsn	0.10	0.09	0.11	0.10	0.13	0.10	0.04	0.07	0.04	0.02	-0.24	0.19	1.00
Sam	0.10	0.08	0.12	0.13	0.14	0.11	0.04	0.09	0.07	0.03	0.08	-1.02	1.00
Off Diag Col Sum:	1.18	1.34	1.31	1.35	1.44	0.92	0.84	0.92	0.91	0.32	0.45	1.03	

Rescale columns by dividing every element in the column by its column sum from the table above:

	Bud	BudL	Mich	MichL	MGD	MGDL	MIL	Coors	CoorsL	Hein	Molsn	Sam	Off Diag Row Sum
Bud	-0.28	0.11	0.12	0.07	0.10	0.05	0.07	0.11	0.06	0.25	0.12	0.05	1.11
BudL	0.12	-0.23	0.09	0.11	0.07	0.08	0.11	0.08	0.10	0.26	0.08	0.05	1.14
Mich	0.15	0.08	-0.38	0.12	0.10	0.05	0.07	0.13	0.06	0.06	0.08	0.09	0.98
MichL	0.10	0.14	0.11	-0.42	0.05	0.09	0.12	0.06	0.13	0.04	0.07	0.08	0.99
MGD	0.12	0.08	0.11	0.06	-0.36	0.16	0.08	0.12	0.07	0.04	0.07	0.09	1.01
MGDL	0.06	0.09	0.05	0.08	0.15	-0.80	0.13	0.07	0.11	0.04	0.10	0.10	0.97
MIL	0.06	0.12	0.06	0.12	0.08	0.13	-0.32	0.07	0.15	0.05	0.05	0.04	0.94
Coors	0.12	0.07	0.12	0.06	0.12	0.06	0.08	-0.50	0.12	0.04	0.08	0.09	0.94
CoorsL	0.04	0.13	0.06	0.13	0.07	0.10	0.16	0.11	-0.32	0.06	0.04	0.07	0.96
Hein	0.07	0.06	0.09	0.09	0.07	0.06	0.08	0.08	0.08	-0.18	0.12	0.16	0.96
Molsn	0.08	0.06	0.08	0.07	0.09	0.10	0.05	0.07	0.05	0.07	-0.54	0.19	0.94
Sam	0.09	0.06	0.09	0.09	0.10	0.12	0.05	0.10	0.08	0.10	0.19	-0.99	1.06
Off Diag Col Sum:	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	

Appendix B - Iterative Rescaling of Cross Elasticity Matrix to Similarities Matrix (cont)

Rescale rows by dividing every element in the row by its row sum from the table above:

	Bud	BudL	Mich	MichL	MGD	MGDL	MIL	Coors	CoorsL	Hein	Molsn	Sam	Off Diag Row Sum
Bud	-0.25	0.10	0.11	0.06	0.09	0.04	0.07	0.10	0.05	0.22	0.11	0.05	1.00
BudL	0.10	-0.20	0.08	0.10	0.06	0.07	0.09	0.07	0.09	0.23	0.07	0.04	1.00
Mich	0.15	0.08	-0.39	0.12	0.10	0.05	0.07	0.13	0.06	0.06	0.08	0.09	1.00
MichL	0.10	0.14	0.11	-0.43	0.05	0.09	0.12	0.06	0.13	0.04	0.07	0.08	1.00
MGD	0.11	0.08	0.11	0.06	-0.36	0.16	0.08	0.12	0.07	0.04	0.07	0.09	1.00
MGDL	0.06	0.09	0.05	0.08	0.15	-0.82	0.13	0.07	0.11	0.04	0.11	0.10	1.00
MIL	0.07	0.13	0.07	0.13	0.08	0.14	-0.34	0.08	0.16	0.05	0.05	0.04	1.00
Coors	0.13	0.07	0.13	0.06	0.12	0.06	0.08	-0.53	0.13	0.04	0.09	0.09	1.00
CoorsL	0.05	0.13	0.06	0.13	0.07	0.11	0.17	0.12	-0.33	0.06	0.04	0.07	1.00
Hein	0.07	0.07	0.09	0.09	0.08	0.06	0.08	0.08	0.08	-0.19	0.12	0.17	1.00
Molsn	0.09	0.07	0.09	0.08	0.10	0.11	0.06	0.08	0.05	0.08	-0.57	0.20	1.00
Sam	0.08	0.06	0.09	0.09	0.09	0.11	0.05	0.09	0.07	0.09	0.18	-0.94	1.00
Off Diag Col Sum:	1.01	1.02	1.00	1.00	1.01	1.01	1.00	1.00	1.01	0.95	0.99	1.02	

Rescale columns by dividing every element in the column by its column sum from the table above:

	Bud	BudL	Mich	MichL	MGD	MGDL	MIL	Coors	CoorsL	Hein	Molsn	Sam	Off Diag Row Sum
Bud	-0.25	0.10	0.11	0.06	0.09	0.04	0.07	0.10	0.05	0.23	0.11	0.04	1.01
BudL	0.10	-0.20	0.08	0.10	0.06	0.07	0.09	0.07	0.09	0.24	0.07	0.04	1.01
Mich	0.15	0.08	-0.39	0.12	0.10	0.05	0.07	0.13	0.06	0.06	0.08	0.09	1.00
MichL	0.10	0.14	0.11	-0.43	0.05	0.09	0.12	0.06	0.13	0.04	0.07	0.08	1.00
MGD	0.11	0.08	0.11	0.06	-0.36	0.16	0.08	0.12	0.07	0.04	0.08	0.09	1.00
MGDL	0.06	0.09	0.05	0.08	0.15	-0.82	0.13	0.07	0.11	0.04	0.11	0.10	1.00
MIL	0.07	0.13	0.07	0.13	0.08	0.13	-0.34	0.08	0.16	0.05	0.05	0.04	1.00
Coors	0.13	0.07	0.13	0.06	0.12	0.06	0.08	-0.53	0.13	0.04	0.09	0.09	1.00
CoorsL	0.05	0.13	0.06	0.13	0.07	0.11	0.17	0.12	-0.33	0.06	0.04	0.07	1.00
Hein	0.07	0.07	0.10	0.09	0.08	0.06	0.08	0.08	0.08	-0.20	0.12	0.16	1.00
Molsn	0.09	0.07	0.09	0.08	0.10	0.11	0.06	0.08	0.05	0.08	-0.58	0.20	1.00
Sam	0.08	0.06	0.09	0.09	0.09	0.11	0.05	0.10	0.07	0.10	0.18	-0.91	1.00
Off Diag Col Sum:	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	

Rescale rows by dividing every element in the row by its row sum from the table above (convergence achieved):

	Bud	BudL	Mich	MichL	MGD	MGDL	MIL	Coors	CoorsL	Hein	Molsn	Sam	Off Diag Row Sum
Bud	-0.25	0.10	0.11	0.06	0.09	0.04	0.06	0.10	0.05	0.23	0.11	0.04	1.00
Budl	0.10	-0.20	0.08	0.09	0.06	0.07	0.09	0.07	0.09	0.24	0.07	0.04	1.00
Mich	0.15	0.08	-0.39	0.12	0.10	0.05	0.07	0.13	0.06	0.06	0.08	0.09	1.00
MichL	0.10	0.14	0.12	-0.43	0.05	0.09	0.12	0.06	0.13	0.04	0.07	0.08	1.00
MGD	0.11	0.08	0.11	0.06	-0.36	0.16	0.08	0.12	0.07	0.04	0.08	0.09	1.00
MGDL	0.06	0.09	0.05	0.08	0.15	-0.82	0.13	0.07	0.11	0.05	0.11	0.10	1.00
MIL	0.07	0.13	0.07	0.13	0.08	0.13	-0.34	0.08	0.16	0.05	0.05	0.04	1.00
Coors	0.13	0.07	0.13	0.06	0.12	0.06	0.08	-0.54	0.13	0.04	0.09	0.09	1.00
CoorsL	0.05	0.13	0.06	0.13	0.07	0.11	0.17	0.12	-0.33	0.06	0.04	0.07	1.00
Hein	0.07	0.07	0.10	0.09	0.08	0.06	0.08	0.08	0.08	-0.20	0.12	0.16	1.00
Molsn	0.09	0.07	0.09	0.08	0.10	0.11	0.06	0.08	0.05	0.08	-0.58	0.20	1.00
Sam	0.08	0.06	0.09	0.09	0.09	0.11	0.05	0.09	0.07	0.09	0.18	-0.91	1.00
Off Diag Col Sum:	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	

Appendix B - Iterative Rescaling of Cross Elasticity Matrix to Similarities Matrix (cont)

Average corresponding off-diagonal elements:

	Bud	BudL	Mich	MichL	MGD	MGDL	MIL	Coors	CoorsL	Hein	Molsn	Sam
Bud	-0.25	0.10	0.13	0.08	0.10	0.05	0.07	0.11	0.05	0.15	0.10	0.06
BudL	0.10	-0.20	0.08	0.12	0.07	0.08	0.11	0.07	0.11	0.15	0.07	0.05
Mich	0.13	0.08	-0.39	0.12	0.11	0.05	0.07	0.13	0.06	0.08	0.09	0.08
MichL	0.08	0.12	0.12	-0.43	0.05	0.09	0.13	0.06	0.13	0.07	0.07	0.08
MGD	0.10	0.07	0.11	0.05	-0.36	0.15	0.08	0.12	0.07	0.06	0.09	0.09
MGDL	0.05	0.08	0.05	0.09	0.15	-0.82	0.13	0.06	0.11	0.05	0.11	0.11
MIL	0.07	0.11	0.07	0.13	0.08	0.13	-0.34	0.08	0.16	0.07	0.05	0.04
Coors	0.11	0.07	0.13	0.06	0.12	0.06	0.08	-0.54	0.12	0.06	0.08	0.09
CoorsL	0.05	0.11	0.06	0.13	0.07	0.11	0.16	0.12	-0.33	0.07	0.05	0.07
Hein	0.15	0.15	0.08	0.07	0.06	0.05	0.07	0.06	0.07	-0.20	0.10	0.13
Molsn	0.10	0.07	0.09	0.07	0.09	0.11	0.05	0.08	0.05	0.10	-0.58	0.19
Sam	0.06	0.05	0.09	0.08	0.09	0.11	0.04	0.09	0.07	0.13	0.19	-0.91

Divide each element by the square root of the product of the two corresponding diagonal elements to get Final Similarity Matrix:

	Bud	BudL	Mich	MichL	MGD	MGDL	MIL	Coors	CoorsL	Hein	Molsn	Sam
Bud	-1.00	0.45	0.42	0.25	0.34	0.11	0.23	0.31	0.17	0.68	0.26	0.13
BudL	0.45	-1.00	0.29	0.40	0.26	0.20	0.42	0.21	0.42	0.76	0.21	0.11
Mich	0.42	0.29	-1.00	0.29	0.29	0.09	0.19	0.29	0.17	0.28	0.18	0.13
MichL	0.25	0.40	0.29	-1.00	0.14	0.15	0.34	0.12	0.35	0.23	0.15	0.13
MGD	0.34	0.26	0.29	0.14	-1.00	0.29	0.24	0.28	0.20	0.23	0.19	0.16
MGDL	0.11	0.20	0.09	0.15	0.29	-1.00	0.25	0.10	0.21	0.14	0.16	0.12
MIL	0.23	0.42	0.19	0.34	0.24	0.25	-1.00	0.19	0.49	0.26	0.12	0.08
Coors	0.31	0.21	0.29	0.12	0.28	0.10	0.19	-1.00	0.29	0.19	0.15	0.13
CoorsL	0.17	0.42	0.17	0.35	0.20	0.21	0.49	0.29	-1.00	0.29	0.11	0.12
Hein	0.68	0.76	0.28	0.23	0.23	0.14	0.26	0.19	0.29	-1.00	0.30	0.30
Molsn	0.26	0.21	0.18	0.15	0.19	0.16	0.12	0.15	0.11	0.30	-1.00	0.26
Sam	0.13	0.11	0.15	0.13	0.16	0.12	0.08	0.13	0.12	0.30	0.26	-1.00

REFERENCES

Cooper, Lee G. (1988). “Competitive Maps: The Structure Underlying Asymmetric Cross Elasticities.” *Management Science,* vol.34(6) (June), 707-723.

Cooper, Lee G., Akihiro Inoue (1996). “Building Market Structures from Consumer Preferences.” *Journal of Marketing Research,* vol.33 (August), 293-306.

Cooper, Lee G., Daniel Klapper, and Akihiro Inoue (1996). “Competitive-Component Analysis: A New Approach to Calibrating Asymmetric Market-Share Models.” *Journal of Marketing Research*, vol.33 (May), 224-238.

Datoo, Bashir A. (1994). “Measuring Price Elasticity.” *Marketing Research,* vol.6(2) (Spring), 30-34.

Devinney, Timothy M. (1988). *Issues in Pricing.* Lexington: Lexington Books.

Green, Paul E., Frank J. Carmone, and Scott M. Smith (1989). *Multidimensional Scaling: Concept and Applications.* Boston: Allyn & Bacon.

Guiltinan, Joseph P., and Gregory T. Gundlach (1996). “Aggressive and Predatory Pricing: A Framework for Analysis.” *Journal of Marketing*, vol.60 (July), 87-102.

Gupta, Sachin, Pradeep Chintagunta, Anil Kaul, and Dick R. Wittink (1996). “Do Household Scanner Data Provide Representative Inferences from Brand Choices: A Comparison with Store Data.” *Journal of Marketing Research*, vol.33 (November), 383-398.

Hair, Joseph F. Jr., Rolph E. Anderson, Ronald L. Tatham, and William C. Black (1995). *Multivariate Data Analysis.* 4th ed. New Jersey: Prentice-Hall.

Hanssens, Dominique M., Leonard J. Parsons, Randall L. Schultz (1990). *Market Response Models: Econometric and Time Series Analysis.* Boston: Kluwer Academic Publishers.

Johnson, Rich (1997), “*Getting the Most from CBC - Part 2*", Sawtooth Software Technical Paper.

Johnston, J. (1984), *Econometric Methods,* 3rd Edition. New York: McGraw-Hill Publishing Company.

Krishnamurthi, Lakshman, S.P. Raj, and K. Sivakumar (1995). “Unique Inter-Brand Effects of Price on Brand Choice.” *Journal of Business Research,* vol.34, 47-56.

Leury, David A. (1990). “How to Predict Market-Share Sensitivity to Price Changes.” *Journal of Pricing Management*, Summer, 1990.

Mohn, N. Carroll (1995). “Pricing Research for Decision Making.” *Marketing Research,* vol.7(1) (Winter), 11-19.

Monroe, Kent B. (1990). *Pricing: Making Profitable Decisions. 2nd ed.* New York: McGraw-Hill.

Nagle, Thomas T. (1987). *The Strategy & Tactics of Pricing*. New Jersey: Prentice-Hall.

Pilon, Thomas L. (1989). “Discriminant versus Factor Based Perceptual Maps: Practical Considerations.” *Sawtooth Software Conference Proceedings*, 166-182.

Pilon, Thomas L. (1992). “A Comparison of Results Obtained from Alternative Perceptual Mapping Techniques.” *Sawtooth Software Conference Proceedings*, 163-178.

Rao, Vithala R. (1984), “Pricing Research in Marketing: The State of the Art,“ *Journal of Business* (January).

Reibstein, David J., and Hubert Gatignon (1984). “Optimal Product Line Pricing: The Influence of Elasticities and Cross-Elasticities.” *Journal of Marketing Research*, vol.21 (August), 259-267.

Richard, Michael D., Anthony W. Allaway, David Berkowitz, and Giles D’Souza (1996). “Capturing Competitive, Cannibalistic, and Variety-Seeking Influences on Market Share: An Asymmetric Modeling Approach.” *Journal of Applied Business Research*, vol.12(3), 108-119.

Smallwood, Richard (1991), “Using Conjoint Analysis for Price Optimization.” *Sawtooth Software Conference Proceedings*, 157-162.

Wyner, Gordon A., Lois H. Benedetti, and Bart M. Trapp, “Measuring the Quantity and Mix of Product Demand.” *Journal of Marketing*, Winter 1984, 101-109.

SOFTWARE REFERENCES

Ci3 System
Choice Based Conjoint (CBC)
both by
Sawtooth Software, Inc.
502 South Still Road
Sequim, WA 98382
(360) 681-2300

P-STAT version 2.19
by
P-STAT, Inc.
230 Lambertville-Hopeville Rd.
Hopewell, NJ 08525
(609) 466-9200

SPSS Professional Statistics 7.5
SYSTAT 6.0 & 7.0
both by
SPSS, Inc.
444 North Michigan Avenue
Chicago, IL 60611
(312) 329-2400