Driver Analysis:
How to Do It Badly . . .and Well

Sawtooth Software Webinar

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Sawtooth Software
The survey software of choice

The logic of driver analysis

- We measure some overall evaluation or outcome variable
  - Overall satisfaction
  - Intent to return
  - Intent to recommend
  - Overall liking
  - Purchase intent
- We also have respondents rate some number of attributes, the potential “drivers” that contribute to that overall measure
  - Performance ratings (e.g. excellent, very good, good, fair, poor)
  - Agreement ratings (from strongly agree to strongly disagree)
  - Etc.
- We use a statistical model to quantify the relation between the drivers (independent variables, IVs) and the overall (dependent variable, DV)

Agenda

- How to do driver analysis badly
  - Correlation
  - Regression
  - Two faces of a shared problem
- What about factor analysis?
- How to do driver analysis well
  - AOO
  - Relative importance weights
  - Random forests
- Summary
- R code
- Q&A

DOING DRIVER ANALYSIS BADLY

How not to do driver analysis

- Correlation – we look at drivers independently and one at a time to see how each one relates to the overall measure (the dependent variable or DV)
- Regression – we model the drivers all at once and see how each one relates to the overall measure partialling out (holding constant) all the others

Casual dining importances - correlation

Attribute	Correlation
Prompt Greeting	0.391
Cleanliness	0.390
Comfortable Environment	0.394
Attentive Server	0.486
Friendly Server	0.435
Appropriate Pace	0.436
Food Taste	0.543
Food Temperature	0.444
Timely Check	0.387
Reasonably Priced	0.400

Correlations are all pretty similar in size.

DV = Satisfaction with last casual dining experience, N=1,284

Casual dining importances - correlation

Attribute	Correlation	Correlation Squared
Prompt Greeting	0.391	0.153
Cleanliness	0.390	0.152
Comfortable Environment	0.394	0.155
Attentive Server	0.486	0.236
Friendly Server	0.435	0.189
Appropriate Pace	0.436	0.190
Food Taste	0.543	0.295
Food Temperature	0.444	0.197
Timely Check	0.387	0.150
Reasonably Priced	0.400	0.160

Squaring the correlations allows us to interpret them as % of variance shared with the overall measure - and is a better measure of importance.

Casual dining importances – multiple regression

Attribute	Correlation Squared	Regression
Prompt Greeting	0.153	0.051
Cleanliness	0.152	-0.051
Comfortable Environment	0.155	0.004
Attentive Server	0.236	0.222
Friendly Server	0.189	0.006
Appropriate Pace	0.190	0.086
Food Taste	0.295	0.389
Food Temperature	0.197	0.031
Timely Check	0.150	0.010
Reasonably Priced	0.160	0.119

Regression explains 36% of the variation in the DV

The halo effect . . .

- Thorndyke (1920) noticed that evaluators tend to rate things they like higher on all attributes and things they don't like lower on all attributes
  - As a result, attributes' correlations are "too high and too even"
  - This is the "halo effect"

. . . results in multicollinearity

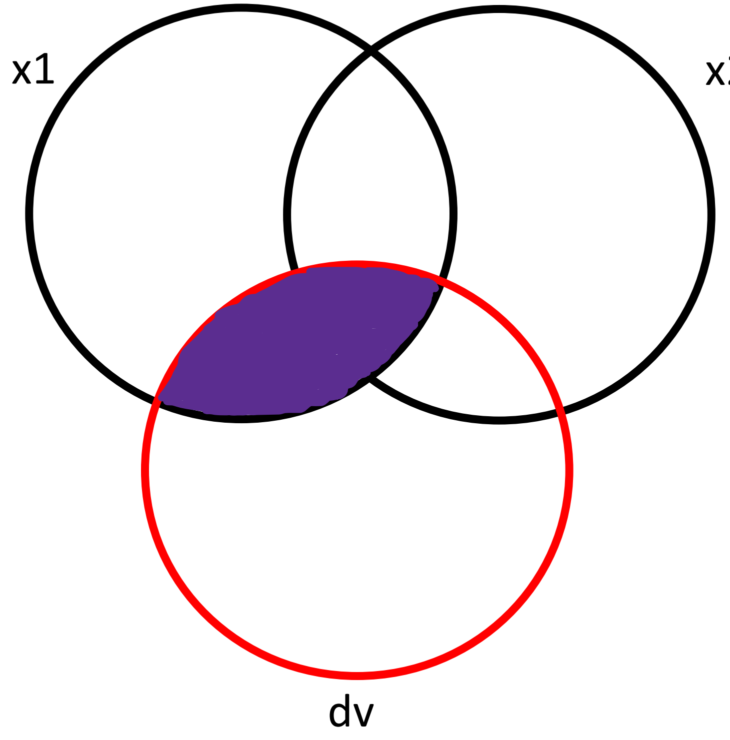
- Each variable is highly correlated with all the others

	Prompt Greeting	Cleanliness	Comfortable Environment	Attentive Server	Friendly Server	Appropriate Pace	Food Taste	Food Temperature	Timely Check	Reasonably Priced
Prompt Greeting	1.00									
Cleanliness	0.58	1.00								
Comfortable Environment	0.52	0.63	1.00							
Attentive Server	0.61	0.55	0.56	1.00						
Friendly Server	0.65	0.61	0.60	0.71	1.00					
Appropriate Pace	0.54	0.58	0.54	0.59	0.57	1.00				
Food Taste	0.48	0.59	0.55	0.55	0.56	0.55	1.00			
Food Temperature	0.54	0.66	0.57	0.61	0.62	0.62	0.63	1.00		
Timely Check	0.52	0.53	0.52	0.64	0.58	0.62	0.46	0.54	1.00	
Reasonably Priced	0.40	0.40	0.48	0.48	0.45	0.49	0.47	0.46	0.45	1.00

- Regression can be a perfectly accurate measure of importance if all off-diagonal cells are 0.00 and it gets less accurate as they are larger than 0.00

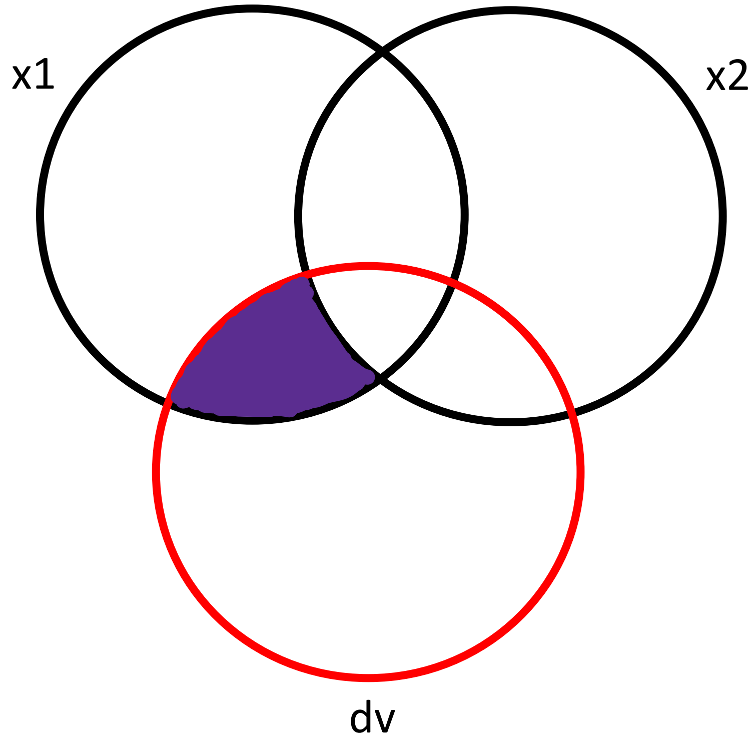
What correlation does

- Credits all variance that overlaps between the overall measure (dv) and an attribute (x1) to that attribute



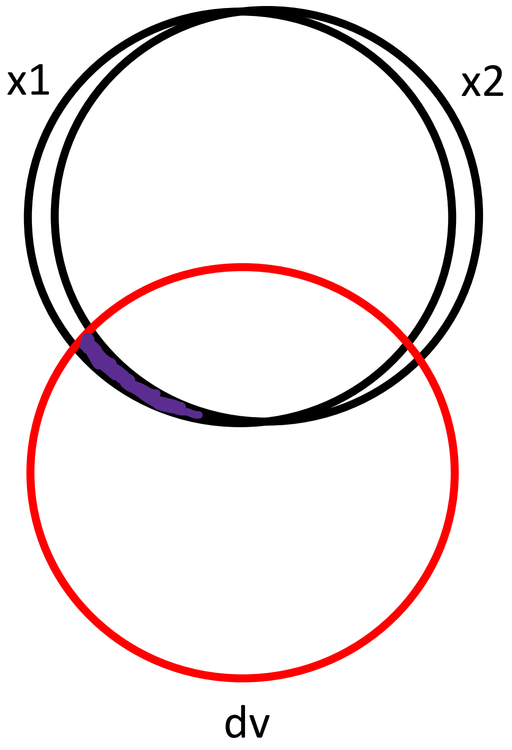
What regression does

- Credits to x1 only the unique variance shared with dv that is not also shared with x2



What multicollinearity does

- When the unique portion of variance is just a thin sliver (because x1 and x2 are highly correlated) small differences in the position of the circles can cause large differences in the size of the sliver (instability)



How bad is our multicollinearity?

- A measure of collinearity is the condition index (c.i.), derived from factor analysis of the predictor variables
  - <5 is good and should produce a useful regression analysis
  - 30+ is considered extreme collinearity and will ruin regression analysis
  - 5-30 is modest collinearity and may be problematic
- The casual dining study had c.i. = 17.6

Casual dining importances - correlation

Attribute	Correlation Squared
Prompt Greeting	0.153
Cleanliness	0.152
Comfortable Environment	0.155
Attentive Server	0.236
Friendly Server	0.189
Appropriate Pace	0.190
Food Taste	0.295
Food Temperature	0.197
Timely Check	0.150
Reasonably Priced	0.160
Variance explained	188%

Correlation double counts importances and explains 188% of the variation in the DV, evidence that the correlations are too high.

Casual dining importances – multiple regression

Attribute	Correlation Squared	Regression
Prompt Greeting	0.153	0.051
Cleanliness	0.152	-0.051
Comfortable Environment	0.155	0.004
Attentive Server	0.236	0.222
Friendly Server	0.189	0.006
Appropriate Pace	0.190	0.086
Food Taste	0.295	0.389
Food Temperature	0.197	0.031
Timely Check	0.150	0.010
Reasonably Priced	0.160	0.119
Variance explained	188%	36%

Regression explains 36% of the variation in the DV.

Regression coefficients show a reversal, which is very common when collinearity is present.

Regression importances

- When using regression for importance measurement, it makes more sense to use squared semi-partial correlations (sr_i^2) than it does to use the regression coefficients themselves (Tabachnick and Fidell 1983)
- These have the further benefit of revealing how much shared variance the regression analysis is counting in R-squared but that it's not attributing to any coefficients

Casual dining importances – multiple regression

Attribute	Correlation Squared	Regression (coefficients)	(sr_i^2)
Prompt Greeting	0.153	0.051	0.0008
Cleanliness	0.152	-0.051	0.000585
Comfortable Environment	0.155	0.004	4.52E-06
Attentive Server	0.236	0.222	0.01226
Friendly Server	0.189	0.006	8.12E-06
Appropriate Pace	0.190	0.086	0.002157
Food Taste	0.295	0.389	0.055465
Food Temperature	0.197	0.031	0.000238
Timely Check	0.150	0.010	2.92E-05
Reasonably Priced	0.160	0.119	0.006777
Variance explained	188%	36%	8%

Regression explains 36% of the variation in the DV, but it discards over three quarters of that information when allocating importance to attributes (28% of the variance is shared among multiple IVs).

Casual dining importances – multiple regression

Attribute	Correlation Squared	Regression (coefficients)	(sr_i^2)	Normalized (sr_i^2)
Prompt Greeting	0.153	0.051	0.0008	0.010
Cleanliness	0.152	-0.051	0.000585	0.007
Comfortable Environment	0.155	0.004	4.52E-06	0.000
Attentive Server	0.236	0.222	0.01226	0.157
Friendly Server	0.189	0.006	8.12E-06	0.000
Appropriate Pace	0.190	0.086	0.002157	0.028
Food Taste	0.295	0.389	0.055465	0.708
Food Temperature	0.197	0.031	0.000238	0.003
Timely Check	0.150	0.010	2.92E-05	0.000
Reasonably Priced	0.160	0.119	0.006777	0.087
Variance explained	188%	36%	8%	8%

It's easier to interpret the squared semi-partial correlations if we normalize them to sum to 100%.

INTERLUDE: WHAT ABOUT FACTOR ANALYSIS?

Factor analysis

- Factor analysis groups the individual attributes into uncorrelated factors, each composed of related attributes
- Factor analysis was developed for testing of psychometric theories
  - If a factor is real, items purported to measure it should load together on a factor - so we can use factor analysis to measure the validity of our conceptual model by verifying theory-driven constructs, e.g.
    - Risk aversion
    - Price sensitivity
    - Need for cognition
    - Conscientiousness
  - If an item is a good measure of a construct, it will load highly on the factor measuring that construct and not highly on any other factor - so we can use factor analysis to test the validity of our measurement model, too

What factor analysis isn’t

- Factor analysis isn’t a garbage can into which we can toss a haphazard assortment of items, with no theorized factor structure, and then rely on the math to sort them out and turn them into valid and useful factors
- Alchemy isn’t real and this is not a picture of “pre-gold”

Factor analysis abuse

- If you treat factor analysis like a garbage can, you may get a garbage result
  - Important items can “cross-load” on several factors and have their impact diluted
  - Important items may not load on any factors, because they don’t share as much variance with the other items as those other items do with each other
- Even if your factor analysis is lovely, interpretation is usually problematic
  - Factors are linear combinations of the items that load on them
  - I’ve yet to meet a marketing manager who cares about whatever factor it is that “ease of use,” “quality” and “timeliness” combine to measure

DOING DRIVER ANALYSIS WELL

Three ways to address multicollinearity

- Slice up the overlap in variance by “averaging over orderings” (AOO)
- Slice up the overlap algebraically - Johnson’s relative importance measure (ε)
- Random Forests – two randomization steps “decorrelate” a forest of decision trees

- The first two above usually produce similar results
- These methods produce ratio-scaled importances that sum to 100%

Averaging over orderings (AOO)

- Imagine a situation with 3 predictors, which can enter a regression model in 6 possible orders
  - abc
  - acb
  - bac
  - bca
  - cab
  - cba
- In two cases a enters first, in two it enters last and in two it enters second, once following b and once following c

Averaging over orderings (AOO)

- If,
  - F is the R^2 for a predicting the dependent variable when a enters the model first (i.e. without b and c)
  - B is the R^2 that a adds to a model that already includes b
  - C is the incremental R^2 that a adds to a model that already includes c
  - L is the incremental R^2 that a adds by entering last (after b and c are already in the model)
- a's average contribution to R^2 is

$$AOO = \frac{2F + B + C + 2L}{6}$$

- Repeat this for all attributes and normalize them to sum to 100%

AOO is computationally intensive

- For k predictors there are k! possible orderings
- For example, with k=21 there are 5.1 x 10^{19} orderings – more than there are grains of sand on the earth

- Beyond about 20 predictors, AOO takes a <u>really</u> long time to run

AOO History

- Lindeman, Merenda and Gold (1980) recommended averaging incremental squared semi-partial correlations (i.e. incremental R^2) across all possible orderings
- Cox (1985) also suggested an averaging approach and noted that it was equivalent to the Shapley value
- Kruskal (1987) seems to have been unaware of LMG, coined the term AOO, and suggested using squared partial correlations instead of squared semi-partial correlations
- Theil and Chung (1988) recommend using an information theoretic entity, entropy, as the thing that gets averaged over orderings and Soofi *et al* (2000) note that using entropy allows ANOVA and logit models to fit into the AOO framework
- Budescu's (1993) dominance analysis reproduces LMG results
- Lipovetsky and Conklin (2001) derived LMG from Shapley's work
- Feldman's (2005) proportional marginal variance decomposition (PMVD) uses a weighted average of the elements that go into the LMG calculation

Casual dining importances - AOO

Attribute	Correlation Squared	Regression (coefficients)	Normalized (sr_i^2)	LMG (AOO)
Prompt Greeting	0.153	0.051	0.010	0.061
Cleanliness	0.152	-0.051	0.007	0.056
Comfortable Environment	0.155	0.004	0.000	0.060
Attentive Server	0.236	0.222	0.157	0.142
Friendly Server	0.189	0.006	0.000	0.078
Appropriate Pace	0.190	0.086	0.028	0.087
Food Taste	0.295	0.389	0.708	0.279
Food Temperature	0.197	0.031	0.003	0.087
Timely Check	0.150	0.010	0.000	0.058
Reasonably Priced	0.160	0.119	0.087	0.091
Variance explained	188%	36%	8%	36%

AOO importances sum to 100% and have ratio-level interpretation

Relative importance weights history

- Gibson (1962) and Johnson (1966) suggested deriving orthogonal variables that are as close as possible to the original (survey) variables
- Turns out they were not all that close, however, so Green *et al* (1978) proposed a way of relating the orthogonal variables and the original variables
- Johnson (2000) proposed a better way of relating the original variables and their orthogonal transformations
- The result, which Johnson called ε, is an algebraic solution for dividing up the overlap in variance that works for any number of predictors

Casual dining importances – Johnson's ε

Attribute	Correlation Squared	Regression (coefficients)	Normalized (sr_i^2)	LMG (AOO)	ε
Prompt Greeting	0.153	0.051	0.010	0.061	0.066
Cleanliness	0.152	-0.051	0.007	0.056	0.054
Comfortable Environment	0.155	0.004	0.000	0.060	0.061
Attentive Server	0.236	0.222	0.157	0.142	0.137
Friendly Server	0.189	0.006	0.000	0.078	0.075
Appropriate Pace	0.190	0.086	0.028	0.087	0.089
Food Taste	0.295	0.389	0.708	0.279	0.269
Food Temperature	0.197	0.031	0.003	0.087	0.087
Timely Check	0.150	0.010	0.000	0.058	0.060
Reasonably Priced	0.160	0.119	0.087	0.091	0.103
Variance explained	188%	36%	8%	36%	36%

Random Forests

- Proposed by Brieman (2001)
- Instead of a single regression tree, RF builds a forest of trees
  - Each tree uses only a random subset of respondents and
  - At each branch of each tree, the algorithm considers only a random subset of variables
- These two randomizations "decorrelate" the forest
- RF produces two importance metrics, one of which ("increase in node purity") works better as a measure of relative importance

Casual dining importances – random forests

Attribute	Correlation Squared	Regression (coefficients)	Normalized (sr_i^2)	LMG (AOO)	ε	RF
Prompt Greeting	0.153	0.051	0.010	0.061	0.066	0.061
Cleanliness	0.152	-0.051	0.007	0.056	0.054	0.067
Comfortable Environment	0.155	0.004	0.000	0.060	0.061	0.071
Attentive Server	0.236	0.222	0.157	0.142	0.137	0.143
Friendly Server	0.189	0.006	0.000	0.078	0.075	0.087
Appropriate Pace	0.190	0.086	0.028	0.087	0.089	0.085
Food Taste	0.295	0.389	0.708	0.279	0.269	0.255
Food Temperature	0.197	0.031	0.003	0.087	0.087	0.084
Timely Check	0.150	0.010	0.000	0.058	0.060	0.065
Reasonably Priced	0.160	0.119	0.087	0.091	0.103	0.082
Variance explained	188%	36%	8%	36%	36%	

Significance testing

Attribute	Correlation Squared	Regression (coefficients)	Normalized (sr_i^2)	LMG (AOO)	ε	RF
Prompt Greeting	**0.153**	0.051	0.010	**0.061**	**0.066**	0.061
Cleanliness	**0.152**	-0.051	0.007	**0.056**	**0.054**	0.067
Comfortable Environment	**0.155**	0.004	0.000	**0.060**	**0.061**	0.071
Attentive Server	**0.236**	**0.222**	**0.157**	**0.142**	**0.137**	0.143
Friendly Server	**0.189**	0.006	0.000	**0.078**	**0.075**	0.087
Appropriate Pace	**0.190**	**0.086**	0.028	**0.087**	**0.089**	0.085
Food Taste	**0.295**	**0.389**	**0.708**	**0.279**	**0.269**	0.255
Food Temperature	**0.197**	0.031	0.003	**0.087**	**0.087**	0.084
Timely Check	**0.150**	0.010	0.000	**0.058**	**0.060**	0.065
Reasonably Priced	**0.160**	**0.119**	**0.087**	**0.091**	**0.103**	0.082
Variance explained	188%	36%	8%	36%	36%	

Significant drivers in green

RF importances don't have stat tests available

Correlation, LMG and Johnson's ε are most sensitive in detecting significant drivers

Agreement of methods

- Correlation matrix of importance vectors
- Note LMG and Johnson's epsilon are the most highly correlated, followed by either of them with RF

	Correlation Squared	Regression	Normalized (sr_i^2)	LMG (AOO)	ε	RF
Correlation Squared	1.000					
Regression (coefficients)	0.896	1.000				
Normalized (sr_i^2)	0.872	0.917	1.000			
LMG (AOO)	0.949	0.952	0.979	1.000		
ε	0.933	0.963	0.980	**0.997**	1.000	
RF	0.956	0.936	0.973	0.995	0.987	1.000

How bad was our multicollinearity?

- A measure of collinearity is the condition index, derived from factor analysis of the predictor variables
  - <5 is good and should produce a useful regression analysis
  - 30+ is considered extreme collinearity and will ruin regression analysis
  - 5-30 is questionable
- The casual dining study had C.I. = 17.6
- Let's try 3 more
  - Airline (c.i. = 17.0, n=701)
  - Auto Service (c.i. = 19.6, n=702)
  - Burger (c.i. = 33.8, n=3,202)

Airline importances

Attribute	Correlation Squared	Regression (coefficients)	Normalized (sr_i^2)	LMG (AOO)	ε	RF
Reservation process	0.167	0.023	0.005	0.078	0.080	0.068
Boarding process	0.239	0.132	0.240	0.165	0.169	0.150
Food and beverage service	0.198	0.075	0.092	0.107	0.110	0.117
Luggage policy	0.161	0.020	0.009	0.070	0.072	0.081
Width of my seat	0.192	0.047	0.019	0.086	0.083	0.081
Amount of legroom	0.191	0.013	0.002	0.083	0.078	0.113
Overhead bin space	0.175	0.011	0.001	0.073	0.070	0.078
How staff treated me	0.286	0.228	0.536	0.236	0.232	0.209
Price of the ticket	0.192	0.078	0.096	0.102	0.106	0.102
% of variance	180%	38%	7%	38%	38%	

Auto service

Attribute	Correlation Squared	Regression (coefficients)	Normalized (sr_i^2)	LMG (AOO)	ε	RF
Quality	0.425	0.150	0.102	0.119	0.110	0.143
Ease	0.160	-0.045	0.021	0.027	0.027	0.033
Transportation	0.121	0.044	0.032	0.027	0.035	0.041
Knowledge	0.280	0.124	0.120	0.076	0.087	0.047
First Time	0.392	0.146	0.134	0.109	0.107	0.106
Price	0.248	0.129	0.179	0.072	0.086	0.048
Keep Me Informed	0.291	-0.074	0.033	0.055	0.050	0.039
No Overage	0.284	0.024	0.004	0.058	0.060	0.056
Ready	0.325	0.016	0.002	0.067	0.065	0.046
Explain	0.382	0.095	0.050	0.094	0.090	0.121
Honestly	0.434	0.227	0.286	0.138	0.129	0.166
Understanding	0.364	-0.006	0.000	0.079	0.075	0.086
Timeliness	0.348	0.081	0.038	0.080	0.079	0.066
	405%	56%	6%	56%	56%	56%

Burger joint study

Attribute	Correlation Squared	Regression (coefficients)	Normalized (sr_i^2)	LMG (AOO)	ε	RF
Fast	0.817	0.024	0.003	0.095	0.097	0.030
Clean	0.841	0.044	0.007	0.100	0.100	0.121
Safe	0.846	0.149	0.078	0.103	0.102	0.164
Correct	0.826	0.093	0.036	0.098	0.099	0.090
Friendly	0.830	0.018	0.001	0.097	0.098	0.077
Delivery	0.839	0.096	0.037	0.101	0.100	0.090
Taste	0.869	0.416	0.766	0.118	0.112	0.283
Hot	0.839	0.019	0.001	0.100	0.100	0.109
Easy	0.814	-0.008	0.000	0.094	0.095	0.022
Price	0.796	0.098	0.070	0.094	0.098	0.013
	832%	89%	1%	89%	89%	89%

Regression R^2 is 89%, almost all of which overlaps among multiple predictors

Agreement of methods across 4 studies

- Correlations of importance vectors, averaged across studies

	Correlation Squared	Regression	Normalized (sr_i^2)	LMG (AOO)	ε	RF
Correlation Squared	1.000					
Regression (coefficients)	0.763	1.000				
Normalized (sr_i^2)	0.713	0.924	1.000			
LMG (AOO)	0.937	0.913	0.889	1.000		
ε	0.898	0.945	0.911	**0.990**	1	
RF	0.921	0.859	0.830	0.960	0.935	1

Summary

- Common measures of attribute importance are badly flawed
- Correlation and regression can be improved, a little
- Newer methods (AOO, ε, RF) make more sense and show convergent validity
- AOO and ε tend to agree the most
- I recommend using
  - AOO for problems with 20 or fewer predictors
  - Johnson's ε for models with 20+ predictors
  - RF when my predictors have a mix of different scales
- https://www.youtube.com/watch?v=_T71D4u9N_A
  - "Don't be someone who uses correlation or regression for driver analysis"
  - "Do be someone who does driver analysis with AOO, RF or ε"

Bonus stuff!

Q: What about predictors with a negative relationship to the dependent variable?

A: If a predictor has a negative *correlation* with a dependent variable, sometimes we'll denote that by showing the importance in red, but sometimes we just reverse the attribute label instead

Q: What if I don't have access to R?

A: You can do Johnson's epsilon by uploading your data to
https://relativeimportance.davidson.edu/multipleregression.html

Q: How do I run these methods in R?

A: See the remaining slides

R code - correlation

```
setwd("C://Users//keith//Documents//Presentations//2020//Driver analysis webinar//1284")
getwd()


# first row contains variable names, comma is separator
# assign the variable id to row names
# note the / instead of \ on mswindows systems

df<-read.csv("Final1284.csv", header=TRUE)
attach(df)

# Correlation matrix
my_data <- df[, c(2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12)]
res <- cor(my_data)
round(res, 4)
```

R code - regression

```
# Multiple Linear Regression Example
fit <- lm(dv ~ x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10, data=df)
summary(fit) # show results
coefficients(fit) # model coefficients


# semi-partial correlation between "dv" and "x1" given "x2"-"x10"
library(ppcor)
spcor.test(df$dv,df$x1,df[,c("x2","x3","x4","x5","x6","x7","x8","x9","x10")])
```

R code - LMG

```
#Averaging-over-orderings
library(relaimpo)
lmModC <- lm(dv ~ x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10, data = df)
relImportanceC <- calc.relimp(lmModC, type = "lmg", rela = TRUE)
lmModC
relImportanceC
```

R code – Johnson’s epsilon

```
#JOHNSON’S EPSILON, FOR ANY NUMBER OF PREDICTORS, FROM A CORRELATION MATRIX

library(iopsych)

Rs <- matrix(c(1,0.39,0.39,0.39,0.49,0.43,0.44,0.54,0.44,0.39,0.40,
0.39,1,0.58,0.52,0.61,0.65,0.54,0.48,0.54,0.52,0.40,
0.39,0.58,1,0.63,0.55,0.61,0.58,0.59,0.66,0.53,0.40,
0.39,0.52,0.63,1,0.56,0.60,0.54,0.55,0.57,0.52,0.48,
0.49,0.61,0.55,0.56,1,0.71,0.59,0.55,0.61,0.64,0.48,
0.43,0.65,0.61,0.60,0.71,1,0.57,0.56,0.62,0.58,0.45,
0.44,0.54,0.58,0.54,0.59,0.57,1,0.55,0.62,0.62,0.49,
0.54,0.48,0.59,0.55,0.55,0.56,0.55,1,0.63,0.46,0.47,
0.44,0.54,0.66,0.57,0.61,0.62,0.62,0.63,1,0.54,0.46,
0.39,0.52,0.53,0.52,0.64,0.58,0.62,0.46,0.54,1,0.45,
0.40,0.40,0.40,0.48,0.48,0.45,0.49,0.47,0.46,0.45,1),11,11)
ys <- 1
xs <- 2:11
relWt(Rs, ys, xs)
```

R code – random forest

```
# Random Forest
library(randomForest)
set.seed(7291)
fit <- randomForest(dv ~ x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10,
   importance =TRUE, na.action=na.roughfix,
   data=df, ntree=1000)
print(fit) # view results
importance(fit) # importance of each predictor
```

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That's it

QUESTIONS?

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